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Study on Bifurcation of H-H Parameters And Its Variants Contribution To Neurology

Research Article

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Abstract

The important application of bifurcation analysis of a system is estimated by the variables of H-H model. Graphical User Interface (GUI) is essential for proper visualization of results and therefore, here, we are discussing some GUI based bifurcation panels. Based on these panels, we can investigate different abnormal disorders for neurological applications.

KeyWords: H-H model, bifurcation, neurology

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Introduction

Neuron is the fundamental unit for transmitting signals in nervous system [1-3]. The biological membrane is also played an important role in many of life's processes [4].Many of these processes are electrical and the different electrical behavior of nerve cells can be measured experimentally. The flow of ions across the membrane is responsible for the production of membrane potential. The mathematical formulation of the function of neuron was given firstly by H-H model [5-8]. The variation of parameters of H-H equations leads to bifurcation which refers to quantitative changes in the solution structure of dynamical systems.

Neuroscience computes different problems based on neural model from the analysis of bifurcation in H-H variables. This analysis progress in modern quantitative biology and biophysics [9].

It is the main significant issue to control bifurcation because many neuronal disorders are due to bifurcation of neuronal systems. In the field of neurology, these disorders present Alzheimer's disease epilepsy and arrhythmia [10,11]. Bifurcation control has been employed to estimate seizing behavior in the model system of human cortical electrical activity [12].

In this paper, firstly we describe the GUI Morris- Lecar baesd [13] model for global phase portraits. Secondly, we show GUI panel based on the analysis by Jiang Wang [14] that finds the bifurcation would occur when the leakage conductance gl is lower than 0.299406mS/cm^2 . Thirdly, we again show the analysis by Jiang Wang [15] for the investigation of the synchronization of Fitz-Hugh-Nagumo neural system under external electrical stimulation via the nonlinear control by using GUI. Lastly, we discuss the application of H-H variants for contribution in neuroscience based on these example panels.

Analysis of bifurcation in some models

From this table-1, we discuss different bifurcation analysis as follows:

1. GUI Morris- Lecar (ML) model by B.Raesi:

In this ML model, to solve the equation 1, we first need to create functions $m_\infty(v)$ and $n_\infty(v)$. With the help of these functions, we take the different values of different parameters for plotting

Table 1

Scientist' Name	Observation of bifurcation
Fitzhugh-Nagumo	By varying frequency value [16-17]
Jiang Wang	By varying the value of g_l
Morris-lecar	By observing phase portraits classifying as hopf, segment, separator cycle
Hass,Rinzel,Troy	It occurs at the equilibrium of H-H model with a change in the current I_{ext} [18-20]
Rinzel,Miller	Analyzed the stable and unstable solutions of H-H model with variant current I_{ext} and the influence of temperature at the bifurcation point[19]
Guckenheiner, Labourian	By varying Iext and steady state potassium ionic battery V_{k-}[21]
Bedrov and his fellows	By varying sodium conductance g_{Na} and maximal potassium conductance g_k[22]
Matsumoto	Two stable equilibrium potentials coexist in the H-H model by varying I_{ext} , V_k[23]

the various sub panels. Again, with the help of MATLAB library function "ode 23", we solve all the differential equations 2, 3, 4, 5.The solutions are plotted by taking the different values from the table-2. We plot the result of main function capacitive function Cv w. r. t time, n w. r. t time, m w. r. t time respectively. Then the different phase portraits are obtained by plotting the graph by taking three variable Cv, n, m, w. r. t each other as shown in plot (a), plot (b). The equations are shown as:

$C\dot{\upsilon}=i-g_L(\upsilon-E_L)-g_{ca}\, m_\infty(\upsilon-E_{ca})-g_k\, n(\upsilon-E_K)$........(1)

$\dot{n}=\theta\,(n_\infty(\upsilon)-n)/\tau_n$..............(2)

$\dot{m}=\theta\,(m_\infty(\upsilon)-m)/\tau_m$..............(3)

Where

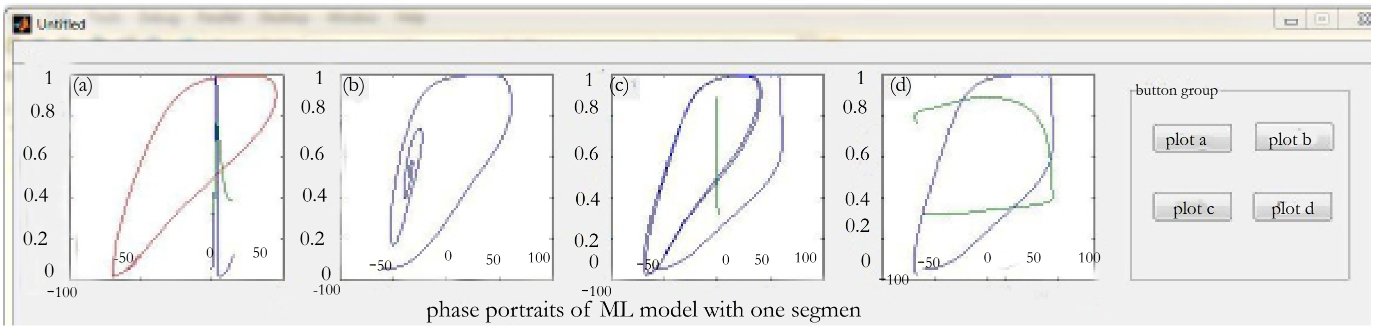
$m_\infty(\upsilon)1/(1+\exp((\upsilon1-1)/(2\upsilon_4)))$ ....(4)

$n_\infty(\upsilon)1/(1+\exp((\upsilon3-1)/(2\upsilon_4)))$ ....(5)

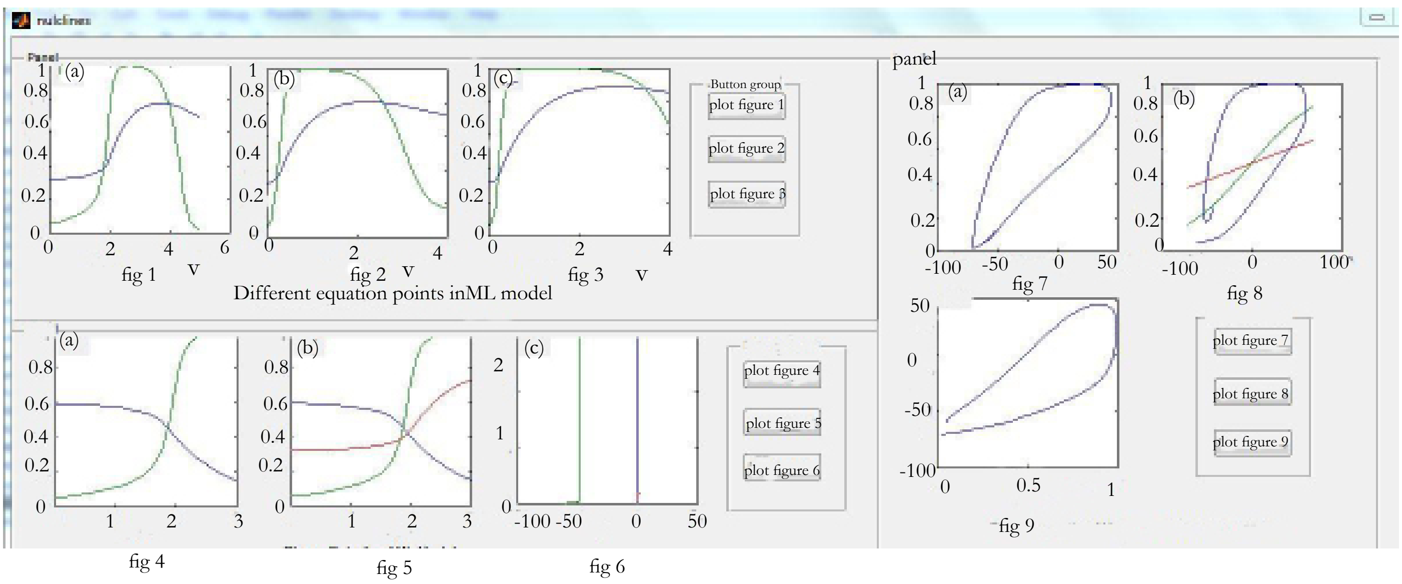
Table 2

fig	i	gl	El	gca	Eca	V1	V2	gk	Ek	ф	V3	V4
Plot(b) fig 1	3	1	-3	7	80	-3	20	-4	-90	1	-30	7
Plot(b) fig 2	3	1	-3	7	80	-3	20	-4	-90	1	-110	60
Plot(b) fig 3	8	7	-79	21	60	-27	11	-10	-90	30	-39	5
Plot(b) fig 4	8	7	-79	21	60	-27	11	10	-90	1	-15	5
Plot(b) fig 5	8	7	-79	21	60	-27	11	10	-90	1	-9.5	5
Plot(b) fig 6	8	7	-79	21	60	-27	11	10	-90	30	-39	5
Plot(a) fig a	0.2	3.25	-42	7	49.5	-10	10	-4.25	-58.2	1	-15	8
Plot(a) fig b	0.2	3.25	-42	7	49.5	-10	10	-4.25	-58.2	1	-10	8
Plot(a) fig c	0.2	3.25	-42	7	49.5	-10	10	-4.25	-58.2	1	-9.3	8
Plot(a) fig d	0.2	3.25	-42	7	49.5	-10	10	-4.25	-58.2	1	-9.5	8
Plot(b) fig 7	21	1	-26	3	75	-31	2	12	78	2	-25	6
Plot(b) fig 8	1	2	-5	4	54	3	3	4	-56	1	-2	6.1
Plot(b) fig 9	9	1	-26	3	75	-30	5	5.9	-78	5.03	-32.6	8

Figure 1
Global phase portraits Panel
plot (a)



Plot (b)



Phase Potraits of ML model

Phase Potraits of ML model with one segment

2. Jiang Wang bifurcation for g_l value:

The HH model can be described using following four equations:

$dv/dt=f_v(V,m,h,n)=1/C_m(I_{ext}-g_{na}m^3h(V-v_{na})-g_k n^4(V-v_k)-g_l(V-v_l))$

$dm/dt= f_m(V,m)=\alpha_m(V)\ (1-m)-\beta_m\ (V)m$

$dh/dt= f_h(V,h)= \alpha_h(V)\ (1-h)-\beta_h(V)h$

$dn/dt= f_n(V,n)= \alpha_n(V)\ (1-n)-\beta_n(V)n$

where, $\alpha_m= 0.08\ (V+56)/(1-\exp(-(V+56)/6.8)$

$\beta_m = 0.8\exp(-(V+56)/18)$

$\alpha_h= 0.006\exp(-(V+41)/14.7)$

$\beta_h= 1.3\ /(1+\exp(-(V+41)/7.6)$

$\alpha_n= 0.0088\ (V+40)/(1-\exp(-(V+40)/7)$

$\beta_n = 0.037\exp(-(V+40)/40)$

Where, V=membrane voltage, Iext =injected current, n=activation variable of potassium channel, m=activation variable of sodium channel, h=inactivation variable of sodium channel, $Cm=1.9\mu F/cm^2$, $g_{na}=50$ mS/ cm², $g_k=22$ mS/ cm², $g_l=0.4$ mS/ cm², $v_{na}=50V$, $v_k=-70V$, $v_l=-81V$.

By putting all the parameter's value, we can find that there is only one bifurcation parameter which is gl..So to analyze the effect the leakage current parameter gl .We did the partial differentiation of above four equations and obtains a Jacobian matrix as shown:

$$J(g1)=\begin{bmatrix} \frac{\partial fv}{\partial V} & \frac{\partial fv}{\partial m} & \frac{\partial fv}{\partial h} & \frac{\partial fv}{\partial n} \\ \frac{\partial fm}{\partial V} & \frac{\partial fm}{\partial V} & 0 & 0 \\ \frac{\partial fh}{\partial V} & 0 & \frac{\partial fh}{\partial V} & 0 \\ \frac{\partial fn}{\partial V} & 0 & 0 & \frac{\partial fn}{\partial V} \end{bmatrix} = \begin{bmatrix} -0.5330303502-0.526315789g1 & 74.10521313 & 612.1467637 & -78.44914264 \\ 0.03888978155 & -1.416022399 & 0 & 0 \\ -0.001512049448 & 0 & -0.3835946452 & 0 \\ 0.002097273245 & 0 & 0 & -0.07833923285 \end{bmatrix}$$

So the coefficients of Jacobian matrix are the solution find the partial differentiation of those equations and putting all values of parameters.

In the nonlinear cable model, the model equation is a coupled differential equation.
We use "ode15" function in MATLAB to solve these equations. After solving the equations the solution is plotted with respect to time having different frequencies as shown in figure-2.

3. Jiang Wang for non-linear model:

The non linear model equations are given as follows with trans-membrane voltage V along the nerve fibre as:

$dX/dt= X(X-1)(1-rY)-Y+I(t)$

$dY/dt=bX$

where

X,Y are membrane voltages, W=recovery variable

vp=peak of action potential

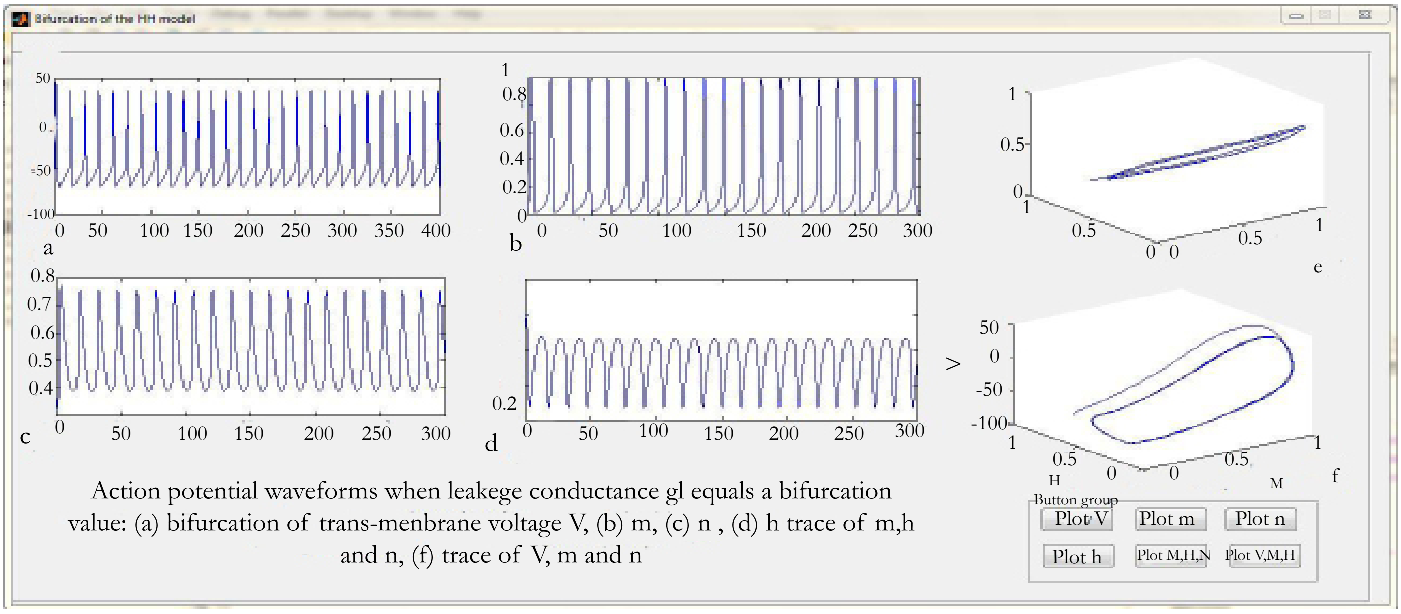
X=V/ vp, Y=W/ vp, r= vp/vT, where, vT=threshold membrane voltage

I(t)=A/w coswt, A=strength of applied field

W=angular frequency of applied field, w=2∏f, f=frequency

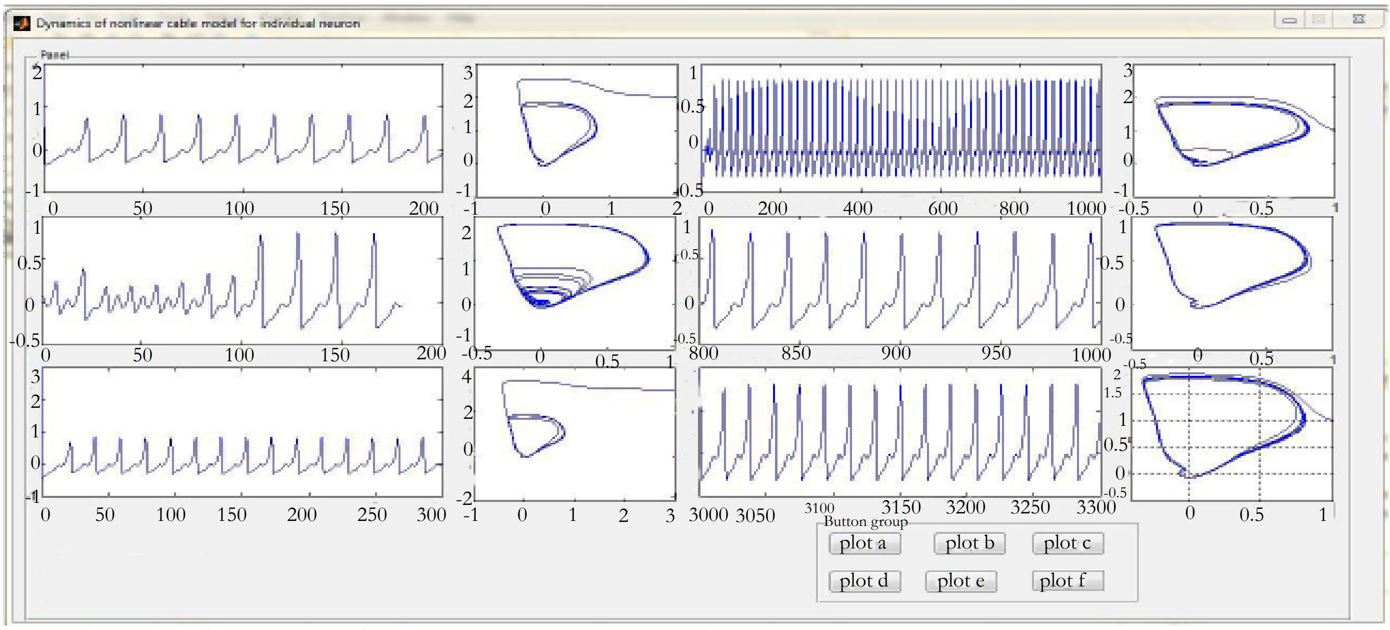
Different observations at different frequency values are concluded with table-3 form the plot figure-3

Figure-2
Jiang Wang bifurcation for g_l value



Action potential waveforms when leakege conductance gl equals a bifurcation value: (a) bifurcation of trans-menbrane voltage V, (b) m, (c) n , (d) h trace of m,h and n, (f) trace of V, m and n

Figure-3
Jiang Wang for non-linear model



The performance of the transmembrane potentials X with various frequency f:
(a)1-periodic oscillation at f=25 Hz_1
(b)quasiperiodic oscillation at f=25.1 Hz_1
(c)1-periodic oscillation again at f=35 Hz_1
the model equation is given bellow:
dX/dt=X(X-1)(1-rX)-Y+l(t)
dY/dt=dX

Table 3

Parameter value	Value of frequency with plots	Observation
r=10, b=1,a=0.1	F=<25Hz (plot a)	Neuron membrane voltage and external electrical driving current oscillates with same frequency
	f>25Hz (plot b)	The system shifts to the quasiperiodic response
	f=35Hz (plot c)	Previous behavior is followed by a sequence of limit cycle
f=variable	f=67Hz (plot d)	A brief region of chaos in encountered
	f=78Hz (plot e)	The response decreases to 2-Periodic oscillation
	f=127.1Hz (plot f)	The system is chaotic again
	f=131Hz	The chaotic oscillation continues

Conclusion

Understanding complex neurobiological systems is one of the most difficult challenges in modern science [24]. From these above results and discussion, we can conclude that H-H equation is the foundation of neuroscience as these parameters values are used for computational brain modeling. It removes ambiguity from theories and makes them logically consistent. Use of computer technology enables theories involving with a large number of elements to be investigated. Computational modeling can help to do the right experiment to solve numerically a set of biologically grounded equations describing the voltage-dependent changes. Computer modeling is an essential component of the neuroscientist's repertoire. Any variation of the H-H parameters can cause bifurcation and this analysis can solve different abnormal disorders by investigating the graphs as shown above. Without H-H model, there is no existence of research in neuroscience as today. So, from these different panels based on H-H equations can solve the problem of investigation of different diseases by researchers also.

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